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METHOD OF OPTIMIZATION OF BLADE SHAPES IN AERODYNAMIC DESIGN OF THE FAN CASCADE

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A method is developed for calculating the geometric parameters of the cascade of an axial fan with a given performance. The method is based on minimization of a certain functional and using the solution of an inverse problem of the flow around the cascade within the framework of the hypothesis of cylindrical cross sections. The functional is minimized by an iterative method of linearization. Examples of calculations are given.

Introduction. Aerodynamic constructions are often imperfect from the viewpoint of exploitation cost, reliability, etc. One method of improving constructions, in particular, fan cascades, is optimization of their geometric parameters. In aerodynamic design of fan cascades, one has to ensure a given performance of the cascade and shock-free input of the flow into the cascade. For this purpose, Batyaev et al. [1, 2] developed a method for calculating the geometric parameters of the cascade, which is based on solving an inverse problem of aerodynamics; the method satisfies additional conditions that allow one to improve the construction performance. In the method proposed, the number of varied geometric parameters equals the number of conditions to be satisfied. Obviously, fixed parameters (those that are not included into the number of varied parameters) form a certain reserve for improving cascade characteristics. In the present work, we propose to use this reserve by means of solving an extreme problem; the minimized functional of this problem can be, for example, expressions for total pressure losses, construction mass, stresses in blades, etc.

1. Formulation of the Problem. We consider the problem of determining the geometric parameters of a double cascade of thin blades of the working wheel of an axial fan (Fig. 1), which should ensure a prescribed performance of the fan with a nonseparated flow of an incompressible fluid around the blades in the so-called basic work regime.

In formulating the problem, we use the condition of shock-free input of the flow into the cascade, and the conditions of a nonseparated flow are taken in the form of inequalities obtained by Dovzhik [3] for plane cascades:

$$D_\varepsilon = (\sin\theta_1/\sin\theta_2)[1.12 + 0.61\sin^2\theta_1(\cot\,\theta_1 - \cot\,\theta_2)/\tau] \leqslant 2. \tag{1.1}$$

Here D_ε is the diffuser parameter of the cascade, τ is the cascade density, and θ_1 and θ_2 are the angles of flow input and output, respectively. These conditions being satisfied, cascade performance is little dependent on fluid viscosity; therefore, the ideal fluid approximation can be used. Under some additional conditions that are not associated with the character of fluid flow, Batyaev et al. [1, 2] developed a method for calculating the geometric parameters of the cascade, which is based on solving an inverse problem of an ideal incompressible fluid flow around the cascade within the framework of the hypothesis of cylindrical cross sections.

In contrast to [4], where methods for solving inverse boundary-value problems were developed, only integral conditions are imposed on the fluid flow in [1, 2]. This approach is related to the parametric definition of profile geometry, commonly accepted in engineering practice, where the number of geometric parameters in a cylindrical cross section of the cascade is limited; in particular, for the cascade type under study, there are 12 parameters (Fig. 2). Some of these parameters are not included into the number of varied quantities in [1, 2]. In the present work, they are used to minimize the functional J. The result of minimization of the functional J is the better

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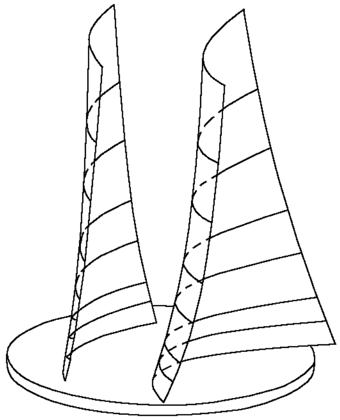

Fig. 1

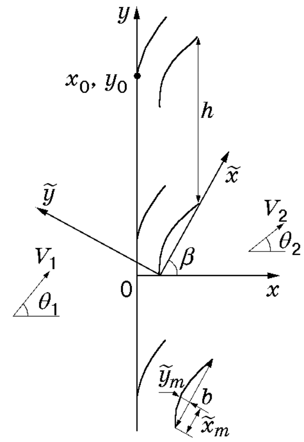

Fig. 2

integral characteristics of the cascade, as compared to the initial ones. The parameters that ensure a minimum of this functional are considered as conventionally optimal.

We introduce the vector $\boldsymbol{x} = \{x_1, \ldots, x_n\}$ whose components are the sought geometric parameters of the cascade (Fig. 2). Here $n = 12N$ (N is the number of cross sections into which the blade cascade is divided). Let $g_i(\boldsymbol{x}) = 0$ $(i = 1, \ldots, m)$ be the imposed conditions and $g_i(\boldsymbol{x}) \leqslant 0$ $(i = m+1, \ldots, m+s)$ be the restrictions of the type (1.1), which should be satisfied for the geometric parameters of the cascade to be determined by the method developed in [1, 2]. Then, we can formulate the following extreme problem:

$$J(\boldsymbol{x}) \longrightarrow \inf, \qquad \boldsymbol{x} \in X; \tag{1.2}$$

$$X = \{\boldsymbol{x} \in E^n\colon \quad g_i(\boldsymbol{x}) = 0,\ i = 1, \ldots, m; \quad g_i(\boldsymbol{x}) \leqslant 0,\ i = m+1, \ldots, m+s\}. \tag{1.3}$$

The functional to be minimized can be, for instance, the total pressure losses on the profile in the flow around the cascade, which are caused by fluid viscosity. In essence, the minimization is directed on increasing the fan efficiency. In the basic flow regime around the cascades, the semi-empirical dependence of the coefficient of profile losses on the maximum bending of the profile mid-line $\bar{f}$ is written as follows [3, 5]:

$$c_x^{\rm pr} = 0.012 + 0.048\bar{f} + 0.0023\tau.$$

With allowance for the hypothesis of cylindrical cross sections, the total pressure losses caused by profile losses are calculated as

$$\Delta P_{\rm pr} = (\rho V_1^2/2) b N c_x^{\rm pr}/(2\pi R), \tag{1.4}$$

where ρ is the fluid density, V_1 is the free-stream velocity, and b is the half-sum of profile-chord lengths in the Rth cross section of the blade.

2. Method of Solving the Optimization Problem. Based on the analysis of a number of numerical methods for solving extreme problems, we chose an iterative method of linearization [6], which is a modified analog of the method of gradient projection and employs all features of the method of solving the inverse problem of cascade aerodynamics proposed in [1, 2]. According to the method of linearization, if the set

$$X_* = \{\boldsymbol{x} \in X\colon \quad J(\boldsymbol{x}) = J_* = \inf J(\boldsymbol{x}) > -\infty\}$$

is not empty, the sequential approximation $\boldsymbol{x}_{k+1}$ of the vector of the sought parameters is determined by solving the following problem:

$$\begin{gathered}\Phi_k(\boldsymbol{x}) = \frac{1}{2}\,|\boldsymbol{x} - \boldsymbol{x}_k|^2 + \beta_k \sum_{j=1}^{n} \frac{\partial J(\boldsymbol{x}_k)}{\partial x_j}\,(x_j - x_{jk}) \longrightarrow \inf, \quad \boldsymbol{x} \in W_k,\ \beta_k > 0,\\ W_k = \Big\{\boldsymbol{x} \in E^n\colon \quad g_i(\boldsymbol{x}_k) + \sum_{j=1}^{n} \frac{\partial g_i(\boldsymbol{x}_k)}{\partial x_j}\,(x_j - x_{jk}) = 0,\ i = 1, \ldots, m;\\ g_i(\boldsymbol{x}_k) + \sum_{j=1}^{n} \frac{\partial g_i(\boldsymbol{x}_k)}{\partial x_j}\,(x_j - x_{jk}) \leqslant 0,\ i = m+1, \ldots, m+s\Big\}\end{gathered} \tag{2.1}$$

(β_k is the step in the method of linearization). The possibility of using this approach to solve the problem of optimization of the geometric parameters of the cascade is conditioned by the fact that the initial direct problem is linear, and the solution $\boldsymbol{x}_k$ of intermediate problems (2.1) does not require a large volume of calculations.

Since the function $\Phi_k(\boldsymbol{x})$ is strongly convex and the set W_k, obviously, is convex and closed, problem (2.1) has a unique solution, according to the theorem of [6]. We introduce the notation

$$\boldsymbol{x} - \boldsymbol{x}_k = \boldsymbol{u}, \qquad \boldsymbol{c} = \beta_k \nabla J(\boldsymbol{x}_k), \qquad \boldsymbol{a}_i = \nabla g_i(\boldsymbol{x}_k), \qquad b_i = -g_i(\boldsymbol{x}_k) \tag{2.2}$$

and rewrite problem (2.1) in a more convenient form:

$$\Phi_k(\boldsymbol{u}) = \langle \boldsymbol{u}, \boldsymbol{u}\rangle/2 + \langle \boldsymbol{c}, \boldsymbol{u}\rangle \longrightarrow \inf; \tag{2.3}$$

$$\boldsymbol{u} \in U_k = \{\boldsymbol{u} \in E^n\colon \quad \langle \boldsymbol{a}_i, \boldsymbol{u}\rangle = b_i,\ i = 1, \ldots, m; \quad \langle \boldsymbol{a}_i, \boldsymbol{u}\rangle \leqslant b_i,\ i = m+1, \ldots, m+s\}. \tag{2.4}$$

Since the set U_k is a polygon, problem (2.3), (2.4) is a problem of quadratic programming. In this case, it is solved using the Lagrangian function and a dual problem to (2.3), (2.4).

The Lagrangian function of problem (2.3), (2.4) has the form

$$L(\boldsymbol{u}, \boldsymbol{\lambda}) = \langle \boldsymbol{u}, \boldsymbol{u}\rangle/2 + \langle \boldsymbol{c}, \boldsymbol{u}\rangle + \langle \boldsymbol{\lambda}, A\boldsymbol{u} - \boldsymbol{b}\rangle = \langle \boldsymbol{u}, \boldsymbol{u}\rangle/2 + \langle \boldsymbol{c} + A^{\mathrm{t}}\boldsymbol{\lambda}, \boldsymbol{u}\rangle - \langle \boldsymbol{\lambda}, \boldsymbol{b}\rangle,$$

where $\boldsymbol{u} \in E^n$, $\boldsymbol{\lambda} \in \Lambda_0 = \{\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_{m+s}) \in E^{m+s}\colon\ \lambda_{m+1} \geqslant 0, \ldots, \lambda_{m+s} \geqslant 0\}$, A is the matrix of dimension $(m+s) \times n$ with rows $\boldsymbol{a}_1, \ldots, \boldsymbol{a}_{m+s}$, and $\boldsymbol{b}$ is the vector with the components $b_1, \ldots, b_{m+s}$.

Since $\Phi_{k*} = \inf \Phi_k(\boldsymbol{u}) > -\infty$, we have $U_{k*} = \{\boldsymbol{u}_* \in U_k\colon\ \Phi_k(\boldsymbol{u}_*) = \Phi_{k*}\} \neq \varnothing$, according to the theorem of [6], the function $L(\boldsymbol{u}, \boldsymbol{\lambda})$ has a saddle point $(\boldsymbol{u}_*, \boldsymbol{\lambda}^*)$, for which, in particular, the following relation is valid:

$$\frac{\partial L(\boldsymbol{u}_*, \boldsymbol{\lambda}^*)}{\partial \boldsymbol{u}} = \boldsymbol{u}_* + \boldsymbol{c} + A^{\mathrm{t}}\boldsymbol{\lambda}^* = 0. \tag{2.5}$$

Here $\boldsymbol{u}_* \in U_{k*}$ and $\boldsymbol{\lambda}^* \in \Lambda_0$ $(i = 1, \ldots, m+s)$. The dual problem to (2.3), (2.4)

$$\psi(\boldsymbol{\lambda}) = \inf_{\boldsymbol{u} \in E^n} L(\boldsymbol{u}, \boldsymbol{\lambda}) \longrightarrow \sup \qquad (\boldsymbol{\lambda} \in \Lambda_0) \tag{2.6}$$

has also a solution.

Owing to the uniqueness of the solution of problem (2.3), (2.4), relation (2.5) yields a unique representation of the solution

$$\boldsymbol{u}_* = -(\boldsymbol{c} + A^{\mathrm{t}}\boldsymbol{\lambda}^*), \tag{2.7}$$

where $\boldsymbol{\lambda}^*$ is an arbitrary solution of problem (2.6). Hence, knowing the expression $\boldsymbol{u}(\boldsymbol{\lambda})$, one can write the function $\psi(\boldsymbol{\lambda})$ explicitly:

$$\psi(\boldsymbol{\lambda}) = L(\boldsymbol{u}(\boldsymbol{\lambda}), (\boldsymbol{\lambda})) = -\langle AA^{\mathrm{t}}\boldsymbol{\lambda}, \boldsymbol{\lambda}\rangle/2 - \langle A\boldsymbol{c} + \boldsymbol{b}, \boldsymbol{\lambda}\rangle - \langle \boldsymbol{c}, \boldsymbol{c}\rangle/2. \tag{2.8}$$

Thus, the dual problem (2.6) of the form

$$-\psi(\boldsymbol{\lambda}) \longrightarrow \inf \qquad (\boldsymbol{\lambda} \in \Lambda_0) \tag{2.9}$$

is also a problem of quadratic programming, but the set Λ_0 has a simpler structure as compared to the set U_k (2.4). Then, knowing some solution $\boldsymbol{\lambda}^*$ of problem (2.9), we obtain the solution of the intermediate optimization problem (2.1) from (2.2) and (2.7):

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{u}_* = \boldsymbol{x}_k - \boldsymbol{c} - A^{\mathrm{t}}\boldsymbol{\lambda}^*. \tag{2.10}$$

Problem (2.9) is significantly simplified if there are no restrictions set by inequalities in the initial problem (1.2), (1.3). Then, we have $s = 0$, and sets (1.3) and (2.4) take the form

$$X = \{\boldsymbol{x} \in E^n\colon \quad g_i(\boldsymbol{x}) = 0,\ i = 1, \ldots, m\}; \tag{2.11}$$

$$U_k = \{\boldsymbol{u} \in E^n\colon \quad \langle \boldsymbol{a}_i, \boldsymbol{u}\rangle = b_i,\ i = 1, \ldots, m\}. \tag{2.12}$$

In this case, $\Lambda_0 = E^m$, and the solution of problem (2.9), with allowance for (2.8), is determined by the system of algebraic equations

$$\frac{d\psi(\boldsymbol{\lambda})}{d\boldsymbol{\lambda}} = AA^{\mathrm{t}}\boldsymbol{\lambda} + A\boldsymbol{c} + \boldsymbol{b} = 0. \tag{2.13}$$

Since $AA^{\mathrm{t}} > 0$, problem (2.9) and, hence, system (2.13) have a unique solution.

TABLE 1

Cascade	$\tilde{y}_{m1}$	β_1^0	β_2^0	$b_1^{\rm bush}$, m	$b_1^{\rm per}$, m
Initial	0.155	61.3	61.8	0.210	0.170
Optimal	0.160	63.3	61.8	0.225	0.143

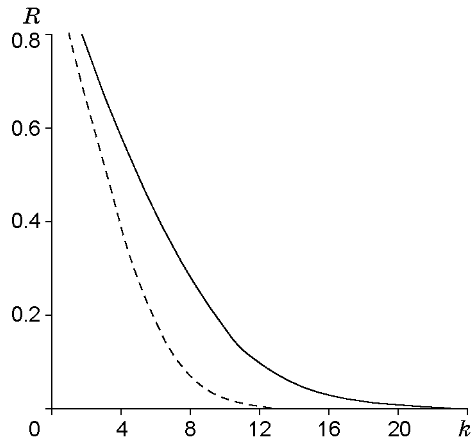

Fig. 3

Thus, for $s = 0$, the solution of problem (1.2), (2.11) reduces to the solution of two systems of linear algebraic equations (2.13) and (2.10) at each iterative step. Since the solution of such systems can be found by a finite number of steps, the solution of the initial problem is also determined by a finite number of iterations. The corresponding coefficients of matrices and vectors of the right sides of these systems are determined by solving the inverse problem of cascade aerodynamics, which was considered in [1, 2].

If there are restrictions in the form of inequalities, the solution of intermediate problems at each iterative step can be reduced to solving a finite number of problems of the type (2.3), (2.12). The possibility of using this approach is related to the finite number of the so-called singular points of problem (2.3), (2.4), which by definition are the solution of problems of the form

$$\Phi_k(\boldsymbol{u}) \longrightarrow \inf, \qquad \boldsymbol{u} \in V_k = \{\boldsymbol{u} \in E^n\colon \quad \langle \boldsymbol{a}_i, \boldsymbol{u}\rangle = b_i,\ i \in \{1,\ldots,m\} \cup I\},$$

where I is an arbitrary subset of indices $\{m+1,\ldots,m+s\}$ (the case $I = \emptyset$ is not excluded). These points are usually determined by special methods of their ordered exhaustive search: a sequence of singular points is constructed, in which the minimized function $J(\boldsymbol{u})$ is rigorously decreasing.

3. Examples of Calculations. To test the program for calculating the optimal shapes of blade cascades, to study the algorithm efficiency, and to determine the rate of convergence of the corresponding iterative process, optimization was performed for the geometric parameters of the cascade of the working wheel of the VO-21 fan, which was designed in 1989 at the Fedorov All-Union Research Institute of Rock Mechanics (Donetsk). The regime corresponding to the performance $Q = 45$ m^3/sec and theoretical total pressure $P_{\rm th} = 6300$ Pa was chosen as the basic work regime. In operating in this regime, the profile losses of this cascade, determined by expression (1.4), are $\Delta P_{\rm pr} = 99.62$ Pa. To solve the optimization problem of minimization of profile losses, the varied parameters additionally included $b_1^{\rm bush}$ (chord length near the bush) and $b_1^{\rm per}$ (chord length at the periphery of the main blade). The chord length of the blades along the radius varied according to a linear law, and the ratio of the chord lengths of the second and main blades was $b_2/b_1 = 0.83$.

A numerical study of the dependence of $\Delta P_{\rm pr}$ on $b_1^{\rm bush}$ and $b_1^{\rm per}$ shows that the profile losses decrease monotonically with decreasing these parameters. At the same time, inequality (1.1) yields the reverse dependence. Thus, according to the method of linearization, the solution of the optimization problem was found by substituting equalities for inequalities (1.1).

By solving the extreme problem, we obtain $\Delta P_{\rm pr} = 94.71$ Pa, which is 5% smaller than for the initial cascade. The values of the varied parameters of the initial and optimal cascades are listed in Table 1.

Figure 3 shows the calculation results for the quantity $R(k) = (J^{(k+1)} - J^{(k)})/J^{(k)}$ characterizing the rate of convergence of successive approximations $J^{(k)} = J(\boldsymbol{x}_k)$ (k is the number of the iteration) in minimization of the functional $J = \Delta P_{\rm pr}$.

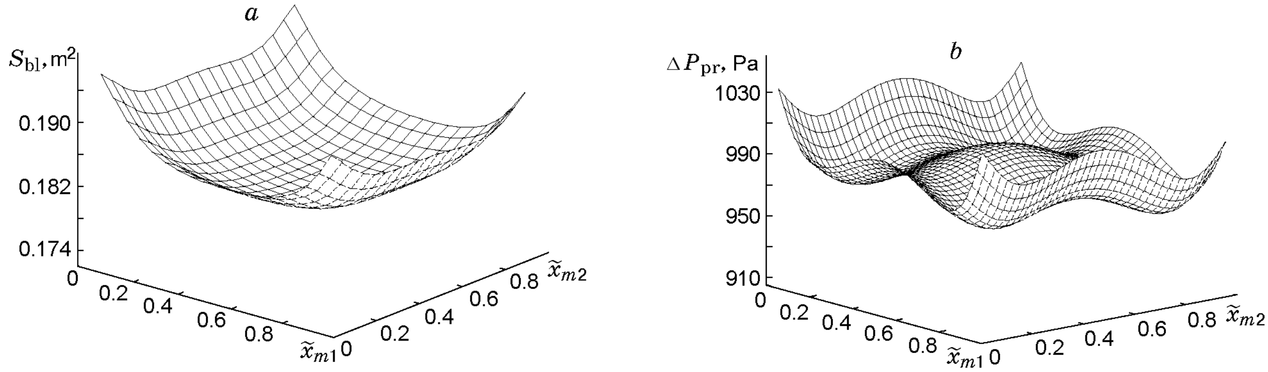

Fig. 4

It should be noted that the rate of convergence of the iterative process depends significantly on the coefficient β_k, which enters the expression for the intermediate optimization problems (2.1). The solid and dashed curves in Fig. 3 refer to $\beta_k = 0.01$ and $k/10$, respectively. A comparison shows a significant advantage of choosing the second value of β_k in terms of convergence of the process. The calculation time for one iteration on a personal computer with a frequency of 715 MHz is approximately 1 sec.

Independent of the solution obtained, the blade shape of this cascade was optimized by minimizing blade areas. As additional varied parameters, we determined the positions of the maximum bending of both blades $\tilde{x}_{m1}$ and $\tilde{x}_{m2}$. The solution yields the values $\tilde{x}_{m1} = \tilde{x}_{m2} = 0.5$ corresponding to chord middles. The optimized parameters allow a 2% decrease in the blade area, which is equivalent to a 150-g decrease in mass of each of the six blades of the wheel for a blade thickness of 5 mm and steel density of 7810 kg/m^3.

Figure 4a shows the calculation results for the blade area $S_{\rm bl}$ obtained by solving inverse problems [1, 2] for various values of $\tilde{x}_{m1}$ and $\tilde{x}_{m2}$ within the range of the chord length 0.1–0.9. Figure 4b shows the calculated dependence of the profile losses of the cascade on the same parameters. A comparison of these dependences shows that the optimal cascade parameters minimizing one of the quantities can be far from being optimal for other quantities. Therefore, in solving the problem of aerodynamic design of cascades and optimizing blade shapes in terms of several parameters, one should determine the priorities and take them into account by introducing appropriate weight coefficients.

Finally, it should be noted that only two additional parameters are included into the number of varied parameters that ensure satisfaction of the basic requirements imposed onto the cascade in the optimization performed, i.e., the optimization is incomplete. To include all geometric parameters of the cascade into the process of global optimization, one has to know the limits of their possible variation caused by engineering and technological conditions.

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